Lecture 9: Forces, Statics and Dynamics

Note: Midterm Exam I, Friday 23rd Sept. Covers Chapters 1-4. 20 questions, 12 conceptual and 8 numerical

Working with forces

1. Choose "body" that you want to work with.
2. Draw all forces on this body.
3. Sum the forces, and apply $\vec{a} = \sum \vec{F}/m$

You need to know that the force of gravity between two masses is given by, $F = \frac{GMm}{r^2}$ and that the weight of a mass, m, is a force given by $w = mg$

The friction force is always in a direction opposite to the direction of the net applied force. The largest force that friction can resist before motion starts is the static friction force $F_s = \mu_s N$, where N is the normal force. Once a body is moving, the friction force is $F_k = \mu_k N$.

Example 1: Consider an Atwood machine, where $m_1 = 5kg, m_2 = 10kg$. Find: a) the acceleration of m_2; b) the tension in the string.

Solution

a) Choose the whole system as the body. Summing the forces and applying Newton's second law, we have,

$$a = \frac{\sum \vec{F}}{m} = \frac{m_2 g - m_1 g}{m_1 + m_2} = 3.27 m/s^2 \quad (down) \tag{1}$$

b) Choose m_2 as the free body. Summing the froces and applying Newton's second law we have,

$$\sum \vec{F} = m_2 g - T = m_2 a \tag{2}$$

Solving for T we find,

$$T = m_2(g - a) = 10kg(9.81 m/s^2 - 3.27 m/s^2) = 65.4N \tag{3}$$

Note that we could do a calculation like this for mass m_1 and find the same value for the tension in the string.

Example 2: Consider a block of mass $m_1 = 5kg$ on a table with static friction coefficient $\mu_s = 0.5$. The block is connected to a second block of mass

m_2 via a massless string over a frictionless pulley. a) Find the largest mass m_2 which the mass m_1 can support without slipping. b) If $m_1 = m_2 = 5kg$ and the coefficient of kinetic friction is $\mu_k = 0.25$, find the acceleration of the masses and the tension in the string.

Solution

a) Apply Newton's second law to the whole system, we have,

$$m_2 g - \mu_s m_1 g = 0 \tag{4}$$

From which we find that $m_2 = \mu_s m_1 = 2.5kg$

b) Once the block is moving, Newton's second law for the whole system states that,

$$m_2 g - \mu_k m_1 g = (m_1 + m_2)a \tag{5}$$

Solving for a, we have,

$$a = \frac{m_2 g - \mu_k m_1 g}{m_1 + m_2} = 3.68m/s^2 \tag{6}$$

To find the tension in the string, apply Newton's second law to either m_1 or to m_2. For m_1, we have,

$$T - \mu_k N = m_1 a, \tag{7}$$

so that,

$$T = \mu_k m_1 g + m_1 a = 30.7N \tag{8}$$

Example 3: Consider a mass, $m = 10kg$, supported by two massless wires attached to a horizontal beam. The first wire makes an angle of 20^0 to the vertical and the second wire makes an angle of -40^0 to the vertical. Find the tensions T_1 and T_2 in the two wires.

Solution

Summing the forces in the x-direction, we have,

$$T_2 Sin(40) - T_1 Sin(20) = 0.64T_2 - 0.34T_1 = 0 \tag{9}$$

which implies that, $T_2 = 0.53T_1$. Summing the forces in the y-direction, we have,

$$T_2 Cos(40) + T_1 Cos(20) - mg = 0 = 0.77(0.53T_1) + 0.94T_1 - 98.1N, \tag{10}$$

where we used, $T_2 = 0.53T_1$. This implies that $T_1 = 73N$, and hence that $T_2 = 39$. Note that $T_1 + T_2 > mg$, why???